Robust Multi-Period Maximum Coverage Facility Location Problem Considering Coverage Reliability

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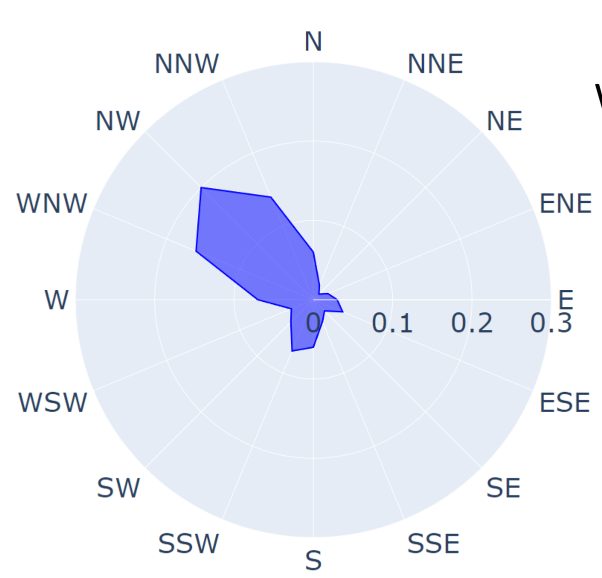
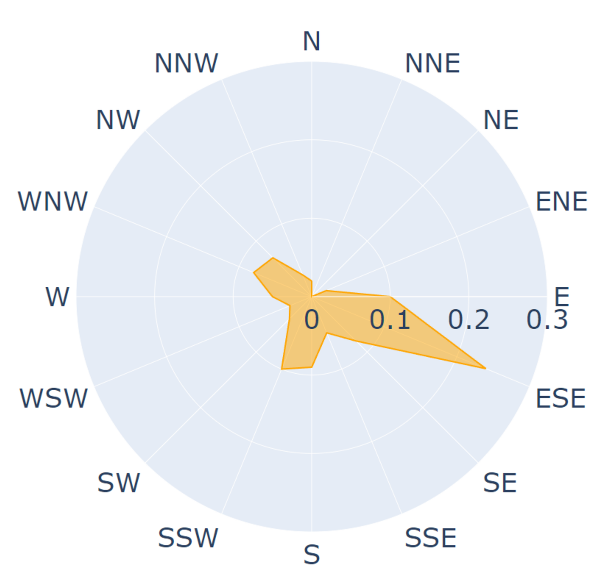
ABSTRACT

- Public service agencies like hospitals, fire, rescue, and police departments are required to maintain high levels of service. These service standards often come as reliability constraints. For example, fire-related incidents require a 90% response rate in 4 minutes.
- Drones or Unmanned Aerial Vehicles (UAVs) are already being site-tested for delivery of automatic external defibrillators (AEDs), medical prescriptions, and medical emergency response as part of federal programs. We consider a case study of tackling out-of-hospital cardiac events using AED-enabled drones in Portland Metro Area, OR-WA.
- Travel time uncertainty in drone deliveries arise from weather conditions, mainly from uncertainty about wind speed and direction [Glick et al. 2021]. The effect of stochasticity in environmental factors is hard to quantify exactly, apart from being data intensive.
- A robust optimization (RO) approach allows for incorporating uncertainty with limited information by using uncertainty sets. Further, splitting of a planning period into multiple smaller periods would disaggregate uncertainties and possibly aid RO in tackling them.
- We develop a compact mixed-integer linear programming formulation of the problem using polyhedral uncertainty sets [Bertsimas and Sim 2004].
- We analyze the value of adding robustness and multiple time periods using a novel Monte-Carlo simulation scheme.



Wind direction distributions at Portland Intl. Airport in Summer and Winter



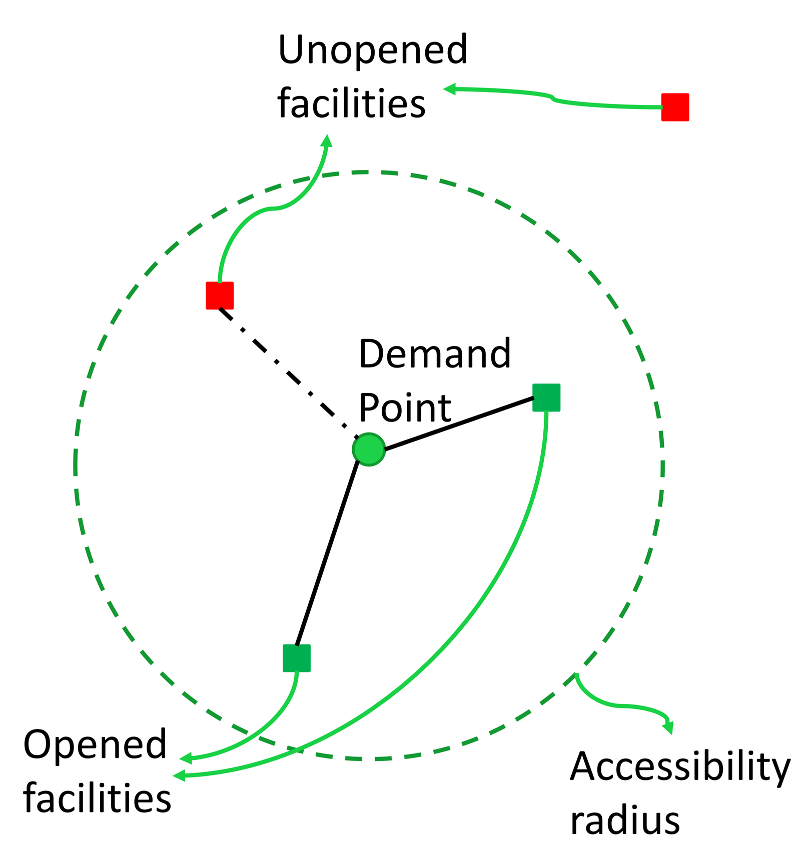
MODELING COVERAGE RELIABILITY

- The service standard constraint is modeled as a chance-constraint to model reliability.
- S_i: set of open facilities that can access demand point i
- p_{ij}: probability of failing to reach demand point i from location j
- a_{ij}: 1, with probability $(1-p_{ij})$, and 0, with probability p_{ij}
- α: reliability standard
- For demand point i to be covered:

$$Prob\left(\sum_{j\in S_i} a_{ij} \geq 1\right) \geq \alpha$$

- Assuming independence among p_{ij}:

$$Prob\left(\sum_{j\in S_i} a_{ij} \geq 1\right) = 1 - \prod_{j\in S_i} p_{ij} \geq \alpha$$



MODEL FORMULATION

- Objective Function:

$$Max_{x,y,z} \sum_{i\in I} c_i x_i$$

- Due to sampling errors stemming from environmental factors like variation in travel time distributions throughout the day, the parameter p_{ij}^t is uncertain. We assume $\bar{p}_{ij}^t$ is the estimate of p_{ij}^t, and $\hat{p}_{ij}^t$ is the maximum deviation of $\bar{p}_{ij}^t$. For our robust model, we assume $p_{ij}^t \in [\bar{p}_{ij}^t - \hat{p}_{ij}^t, \bar{p}_{ij}^t + \hat{p}_{ij}^t]$.
Robust coverage reliability constraint for each demand point $i \in I$ and time period $t \in T$:
$Max_{\{U\subseteq S_i, |U|\leq\Gamma\}} \left[\prod_{j\in U}(\bar{p}_{ij}^t + \hat{p}_{ij}^t)^{y_j^t} \cdot \prod_{j\in S_i\setminus U}(\bar{p}_{ij}^t)^{y_j^t}\right] \leq (1-\alpha)^{x_i}$
- Facility opening constraint for each time period $t \in T$:

$$\sum_{j\in J} y_j^t \leq q$$

- Facility relocation budget constraint and related logical constraints:

$$\sum_{t\in T\setminus\{1\}} \sum_{j\in J} \sum_{k\in J} f_{jk}^t z_{jk}^t \leq B$$

$$\sum_{k\in J} z_{jk}^t = y_j^{t-1} \quad \forall j \in J, t \in T\setminus\{1\}$$

$$\sum_{j\in J} z_{jk}^t = y_k^t \quad \forall k \in J, t \in T\setminus\{1\}$$

- Variable definitions:
$x_i, y_j^t, z_{jk}^t \in \{0,1\}$

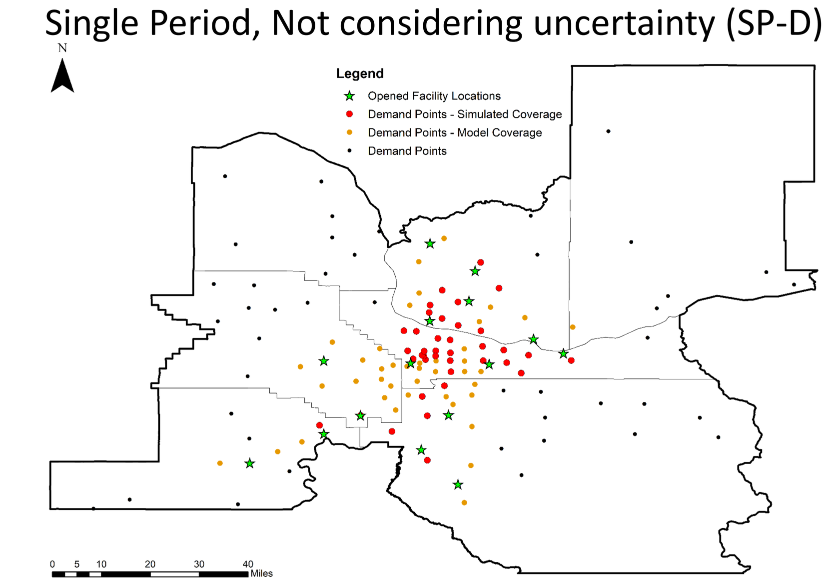
RESULTS AND CONCLUSIONS

- **Description of multiple periods:** We consider a planning period of one whole year. As wind direction distributions are significantly different in Summer (April-September) and Winter (October-March) months, these are considered as different periods. For a single-period formulation, average data of the whole year is utilized.
- Analysis is conducted on four types of models: multi-period robust (MP-R) which consider uncertainty in p_{ij}^t, multi-period deterministic (MP-D) which does not consider uncertainty in p_{ij}^t, single-period robust (SP-R), and single-period deterministic (SP-D).
- **Computational Effort:** Out of 108 models solved for different parameter combinations, 104 converged in 2 hours, and all of them converged in 8 hours. Generally, adding more time periods is more computationally intensive than adding robustness.
- **Value of adding multiple periods and robustness:** Utilizing a multi-period formulation is particularly beneficial when response time thresholds are short, or uncertainty is not accounted for. Adding robustness to deterministic formulations is more beneficial for single-period formulations or when response time thresholds are longer. Combining these different strengths, MP-R improves coverage by 57% compared to SP-D.
- **Geographical impact:** Adding robustness consolidates the facilities towards the densely-populated urban core, thereby improving reliability outcomes.
- **Additional considerations:** Some of the gap between model coverage (coverage promised by the model) and simulated coverage (coverage experienced during Monte-Carlo simulations) can be reduced by either increasing robustness (also reduces model coverage) or increasing the number of opened facilities (has additional associated costs).

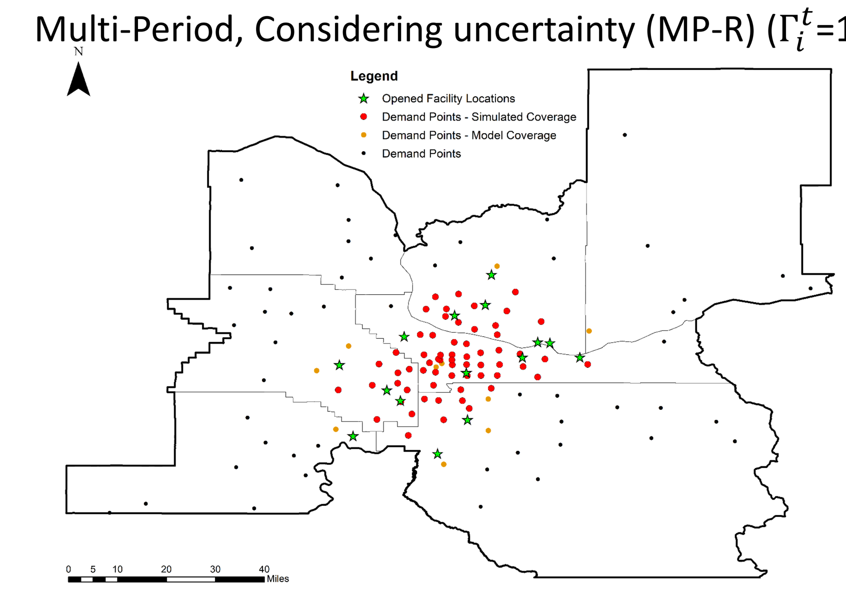
MODEL PERFORMANCE FOR CASE STUDY

- SS1: providing 90% coverage reliability in 4 min; SS2: providing 95% coverage reliability in 10 min
- Improvements in coverage reliability (q=15; SS2)

Single Period, Not considering uncertainty (SP-D)

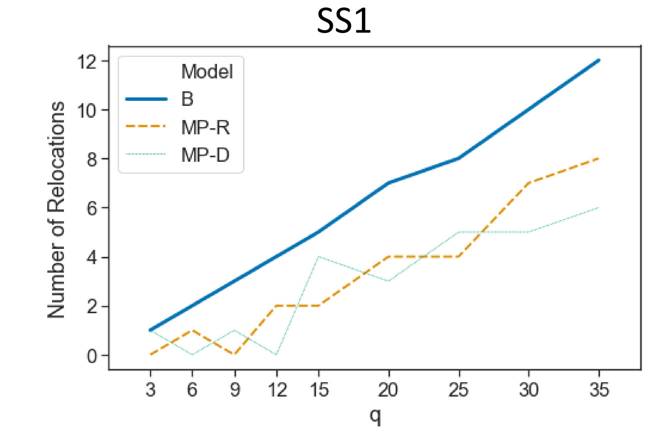


Multi-Period, Considering uncertainty (MP-R) (Γ_i^t=1)

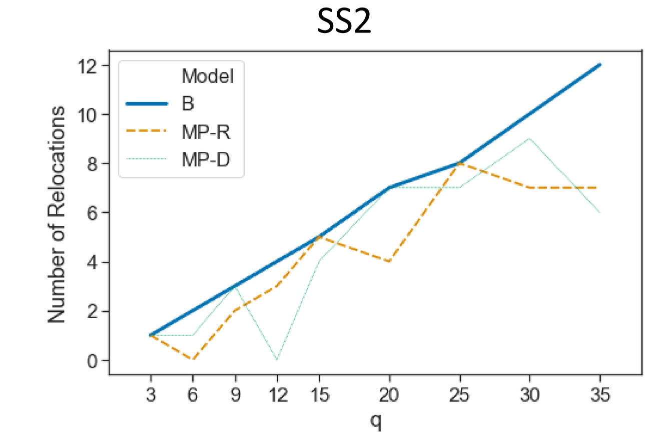


- Extent of facility relocation

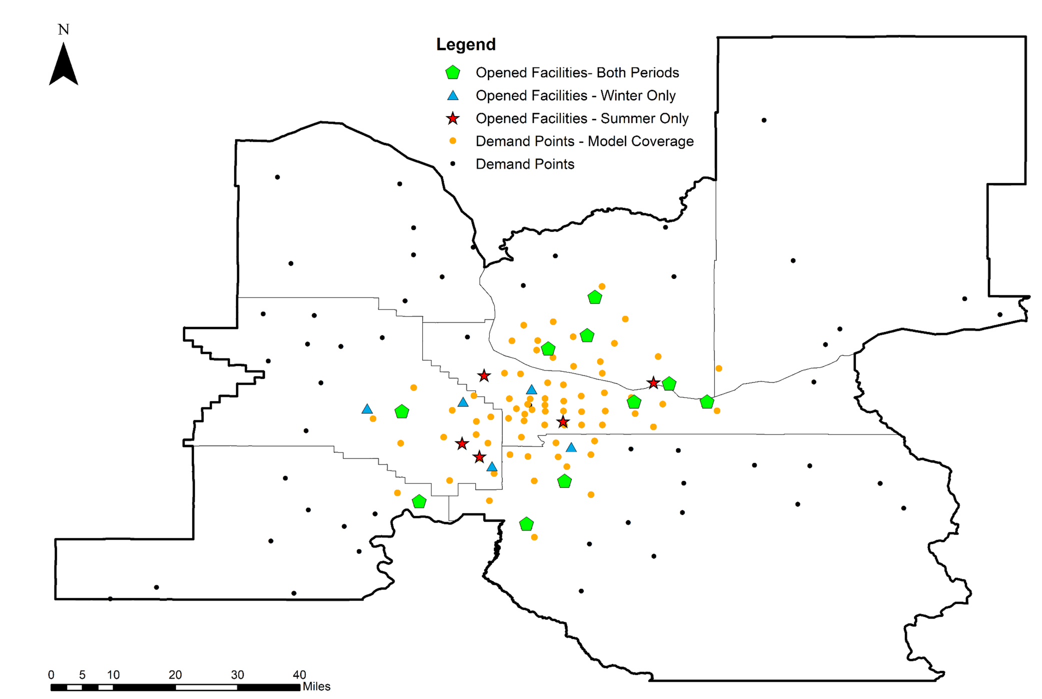
SS1



SS2



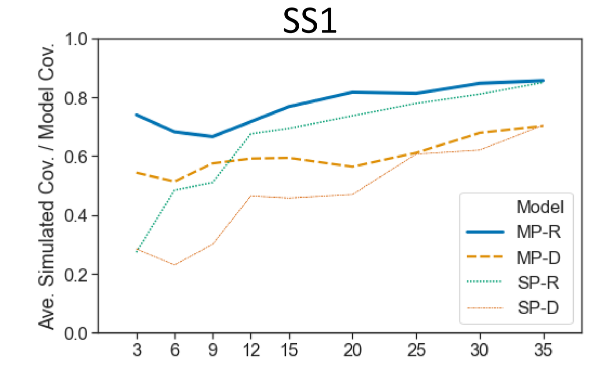
Multi-Period, Considering uncertainty (MP-R) (Γ_i^t=1) (q=15; SS2)



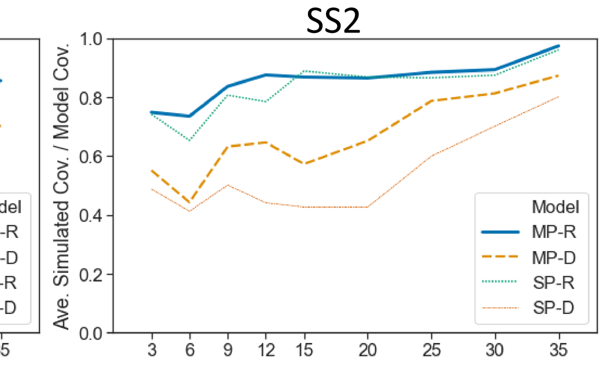
- Reducing the gap between model coverage and simulated coverage

Increasing the number of opened facilities (q) (robust models use Γ_i^t=1)

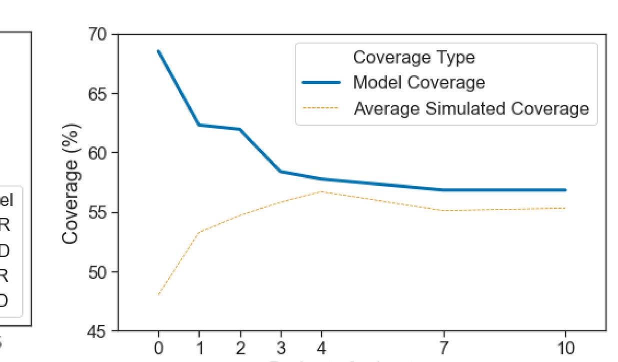
SS1



SS2



Increasing decision conservatism (MP-R; q=35; SS1)



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